

General Certificate of Education  
June 2008  
Advanced Level Examination



**MATHEMATICS**  
**Unit Mechanics 3**

**MM03**

Friday 23 May 2008 9.00 am to 10.30 am

**For this paper you must have:**

- a 12-page answer book
  - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take  $g = 9.8 \text{ m s}^{-2}$ , unless stated otherwise.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 The speed,  $v \text{ m s}^{-1}$ , of a wave travelling along the surface of a sea is believed to depend on

the depth of the sea,  $d \text{ m}$ ,  
the density of the water,  $\rho \text{ kg m}^{-3}$ ,  
the acceleration due to gravity,  $g$ , and  
a dimensionless constant,  $k$

so that

$$v = kd^\alpha \rho^\beta g^\gamma$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are constants.

By using dimensional analysis, show that  $\beta = 0$  and find the values of  $\alpha$  and  $\gamma$ . (6 marks)

- 2 The unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are directed due east and due north respectively.

Two runners, Albina and Brian, are running on level parkland with constant velocities of  $(5\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$  and  $(3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$  respectively. Initially, the position vectors of Albina and Brian are  $(-60\mathbf{i} + 160\mathbf{j}) \text{ m}$  and  $(40\mathbf{i} - 90\mathbf{j}) \text{ m}$  respectively, relative to a fixed origin in the parkland.

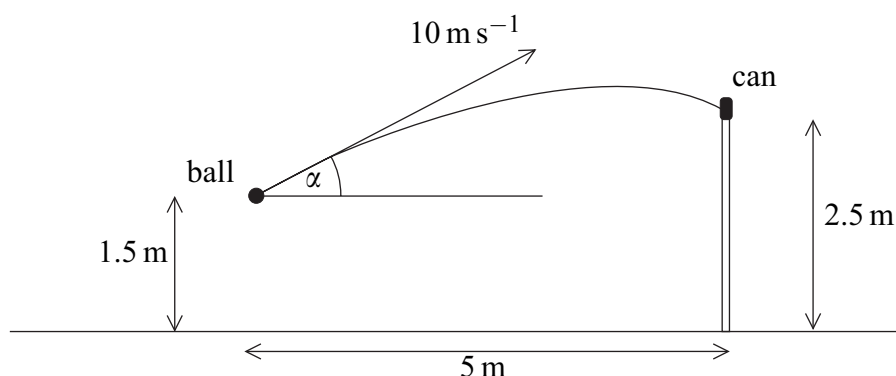
- (a) Write down the velocity of Brian relative to Albina. (2 marks)
- (b) Find the position vector of Brian relative to Albina  $t$  seconds after they leave their initial positions. (3 marks)
- (c) Hence determine whether Albina and Brian will collide if they continue running with the same velocities. (3 marks)

- 3 A particle of mass  $0.2 \text{ kg}$  lies at rest on a smooth horizontal table. A horizontal force of magnitude  $F$  newtons acts on the particle in a constant direction for  $0.1$  seconds. At time  $t$  seconds,

$$F = 5 \times 10^3 t^2, \quad 0 \leq t \leq 0.1$$

Find the value of  $t$  when the speed of the particle is  $2 \text{ m s}^{-1}$ . (4 marks)

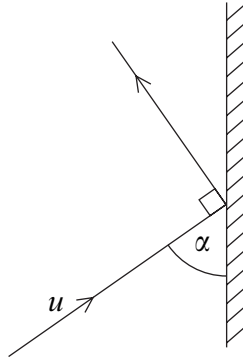
- 4 Two smooth spheres,  $A$  and  $B$ , have equal radii and masses  $m$  and  $2m$  respectively. The spheres are moving on a smooth horizontal plane. The sphere  $A$  has velocity  $(4\mathbf{i} + 3\mathbf{j})$  when it collides with the sphere  $B$  which has velocity  $(-2\mathbf{i} + 2\mathbf{j})$ . After the collision, the velocity of  $B$  is  $(\mathbf{i} + \mathbf{j})$ .
- (a) Find the velocity of  $A$  immediately after the collision. (3 marks)
- (b) Find the angle between the velocities of  $A$  and  $B$  immediately after the collision. (3 marks)
- (c) Find the impulse exerted by  $B$  on  $A$ . (3 marks)
- (d) State, as a vector, the direction of the line of centres of  $A$  and  $B$  when they collide. (1 mark)
- 5 A boy throws a small ball from a height of 1.5 m above horizontal ground with initial velocity  $10 \text{ m s}^{-1}$  at an angle  $\alpha$  above the horizontal. The ball hits a small can placed on a vertical wall of height 2.5 m, which is at a horizontal distance of 5 m from the initial position of the ball, as shown in the diagram.



- (a) Show that  $\alpha$  satisfies the equation
- $$49 \tan^2 \alpha - 200 \tan \alpha + 89 = 0 \quad (7 \text{ marks})$$
- (b) Find the **two** possible values of  $\alpha$ , giving your answers to the nearest  $0.1^\circ$ . (3 marks)
- (c) (i) To knock the can off the wall, the horizontal component of the velocity of the ball must be greater than  $8 \text{ m s}^{-1}$ .
- Show that, for one of the possible values of  $\alpha$  found in part (b), the can will be knocked off the wall, and for the other, it will **not** be knocked off the wall. (3 marks)
- (ii) Given that the can is knocked off the wall, find the direction in which the ball is moving as it hits the can. (4 marks)

- 6 A small smooth ball of mass  $m$ , moving on a smooth horizontal surface, hits a smooth vertical wall and rebounds. The coefficient of restitution between the wall and the ball is  $\frac{3}{4}$ .

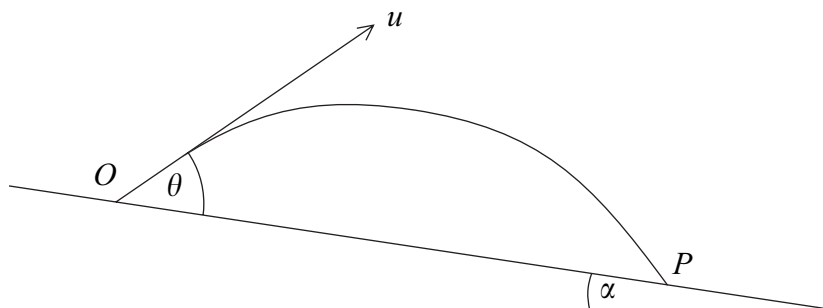
Immediately before the collision, the ball has velocity  $u$  and the angle between the ball's direction of motion and the wall is  $\alpha$ . The ball's direction of motion immediately after the collision is at right angles to its direction of motion before the collision, as shown in the diagram.



- (a) Show that  $\tan \alpha = \frac{2}{\sqrt{3}}$ . (5 marks)
- (b) Find, in terms of  $u$ , the speed of the ball immediately after the collision. (2 marks)
- (c) The force exerted on the ball by the wall acts for 0.1 seconds.

Given that  $m = 0.2 \text{ kg}$  and  $u = 4 \text{ m s}^{-1}$ , find the average force exerted by the wall on the ball. (6 marks)

- 7 A projectile is fired with speed  $u$  from a point  $O$  on a plane which is inclined at an angle  $\alpha$  to the horizontal. The projectile is fired at an angle  $\theta$  to the inclined plane and moves in a vertical plane through a line of greatest slope of the inclined plane. The projectile lands at a point  $P$ , lower down the inclined plane, as shown in the diagram.



- (a) Find, in terms of  $u$ ,  $g$ ,  $\theta$  and  $\alpha$ , the greatest perpendicular distance of the projectile from the plane. (4 marks)
- (b) (i) Find, in terms of  $u$ ,  $g$ ,  $\theta$  and  $\alpha$ , the time of flight from  $O$  to  $P$ . (2 marks)
- (ii) By using the identity  $\cos A \cos B + \sin A \sin B = \cos(A - B)$ , show that the distance  $OP$  is given by  $\frac{2u^2 \sin \theta \cos(\theta - \alpha)}{g \cos^2 \alpha}$ . (6 marks)
- (iii) Hence, by using the identity  $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$  or otherwise, show that, as  $\theta$  varies, the maximum possible distance  $OP$  is  $\frac{u^2}{g(1 - \sin \alpha)}$ . (5 marks)

**END OF QUESTIONS**

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